Geometric Sequences

* exponential

2, 4, 8, 16, ...

×2 ×2 ×2

32, 16, 8, 4, 2,

$r = \frac{1}{2}$

Geometric sequences have a

common ratio.

$$a_n = a_1 r^{n-1}$$

or

$$a_{n+1} = r\, a_n$$

Write the first five terms of the geometric sequence whose first term is 3 and whose common ratio is 2.

$$a_n = 3(2)^{n-1}$$

n	1	2	3	4	5
	3	6	12	24	48

Find the 15th term of the geometric sequence whose first term is 20 and whose common ratio is 1.05

$$a_n = 20(1.05)^{14}$$

$$= 39.596$$

Find the 12th term of the geometric sequence:

5, 15, 45, ...

$\times 3$ $\times 3$

$\frac{45}{15} = 3$

$\frac{15}{5} = 3$

$a_n = 5(3)^{11}$

$= 885{,}735$

The fourth term of a geometric sequence is 125, and the 10th term is 125/64. Find the 14th term.

$125 = a_1 r^3$

$125 r^6 = \frac{125}{64}$

$r^6 = \frac{1}{64}$

$r = \frac{1}{2}$

$\frac{125}{64}\left(\frac{1}{2}\right)^4$

Sum of a Finite Geometric Sequence

$$\sum_{i=1}^{n} a_1 r^{i-1} = a_1\left(\frac{1-r^n}{1-r}\right)$$

Find the sum:

$$\sum_{i=1}^{12} 4(0.3)^{i-1}$$

$$a_1\left(\frac{1-r^n}{1-r}\right)$$

$$4\left(\frac{1-.3^{12}}{1-.3}\right) = 5.71$$

Geometric Series: the summation of an infinite geometric sequence

$$= a_1\left(\frac{1- r^n}{1 - r}\right)$$

$$\sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1 - r}$$

$|r| < 1$

Find the sum:

$$\sum_{n=1}^{\infty} 4(0.6)^{n-1}$$

$$\frac{a_1}{1-r} = \frac{4}{1-.6} = 10$$